SHORE

Examination Number:
Set:

Year 12

Mathematics

Trial HSC Examination

2017

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- NESA-approved calculators may be used
- Answer Questions 1 – 10 on the Multiple Choice Answer Sheet provided
- Start each of Questions 11 – 16 in a new writing booklet
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations
- Write your examination number on the front cover of each booklet
- If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A" on the front cover
- A NESA Reference Sheet is provided.

Total marks – 100

Section I **Pages 1 – 7**

10 marks

- Attempt Questions 1 – 10
- Allow about 10 minutes for this section

Section II **Pages 8 – 15**

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours 45 minutes for this section

Note: Any time you have remaining should be spent revising your answers.

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 What is 54721.35 written in scientific notation, correct to 2 significant figures?

(A) 5.5×10^{-4}

(B) 5.47×10^{4}

(C) 5.5×10^{4}

(D) 5.47×10^{-4}

2 The quadratic equation $2x^2-5x-9=0$ has roots α and β.
What is the value of $\frac{1}{\alpha}+\frac{1}{\beta}$?

(A) $-\frac{9}{2}$

(B) $-\frac{9}{5}$

(C) $-\frac{5}{9}$

(D) $-\frac{5}{2}$

3 Which of the following defines the domain of the function $f(x)=\frac{1}{\sqrt{x+2}}$?

(A) $x\neq-2$

(B) $x\geq-2$

(C) $x<-2$

(D) $x>-2$

4 For the angle θ, $\sin\theta = \frac{5}{13}$ and $\cos\theta < 0$.

Which of the following statements is correct?

(A) $\cos\theta = \frac{12}{13}$

(B) $\operatorname{cosec}\theta = -\frac{13}{12}$

(C) $\tan\theta = -\frac{5}{12}$

(D) $\sec\theta = -\frac{5}{13}$

5 "Although the flood waters are still rising, the increase in the depth of the water will be less today as the showers are clearing."

If the depth of the flood waters is W and the time is t, which of the following statements is true?

(A) $\frac{dW}{dt} > 0,\ \frac{d^2W}{dt^2} > 0.$

(B) $\frac{dW}{dt} > 0,\ \frac{d^2W}{dt^2} < 0.$

(C) $\frac{dW}{dt} < 0,\ \frac{d^2W}{dt^2} > 0.$

(D) $\frac{dW}{dt} < 0,\ \frac{d^2W}{dt^2} < 0.$

6 If $\log_{10} y = 2 - \log_{10} x$, which expression is equivalent to y?

(A) $y = \log_{10} 2 - x$

(B) $y = \frac{100}{x}$

(C) $y = 2 - x$

(D) $y = 100 - x$

7 Joshua plays a video game three times. The probability that he wins at least one game is $\frac{37}{64}$.

What is the probability of Joshua **winning** one game?

(A) $\frac{1}{4}$

(B) $\frac{3}{4}$

(C) $\frac{1}{64}$

(D) $\frac{27}{64}$

8 Using Simpson's Rule with 2 applications, which expression gives the approximate area under the curve $y = \log_e x$ between $x = 1$ and $x = 2$?

(A) $\frac{1}{12}\left[\ln 2 + 2\ln 1.5 + 4(\ln 1.25 + \ln 1.75)\right]$

(B) $\frac{1}{12}\left[\ln 2 + 2(\ln 1.25 + \ln 1.75) + 4\ln 1.5\right]$

(C) $\frac{1}{48}\left[\ln 2 + 2\ln 1.5 + 4(\ln 1.25 + \ln 1.75)\right]$

(D) $\frac{1}{48}\left[\ln 2 + 2(\ln 1.25 + \ln 1.75) + 4\ln 1.5\right]$

9 The graph of $y = f(x)$ is shown below.

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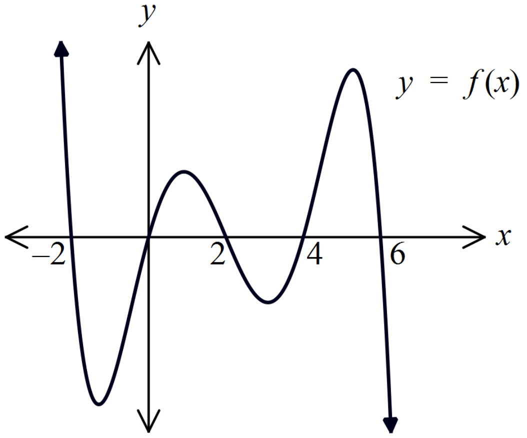
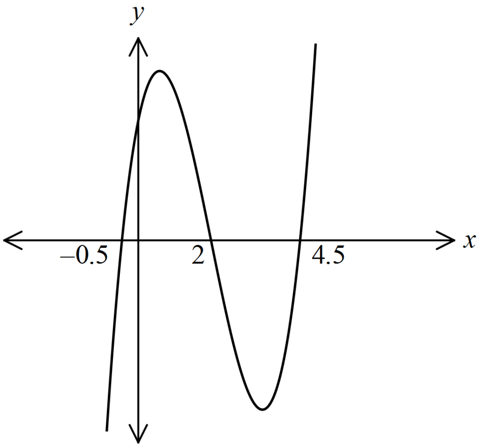
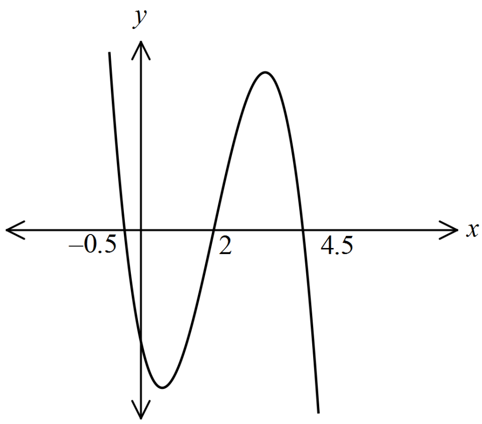


Which of these graphs could represent the gradient function $y = f'(x)$?

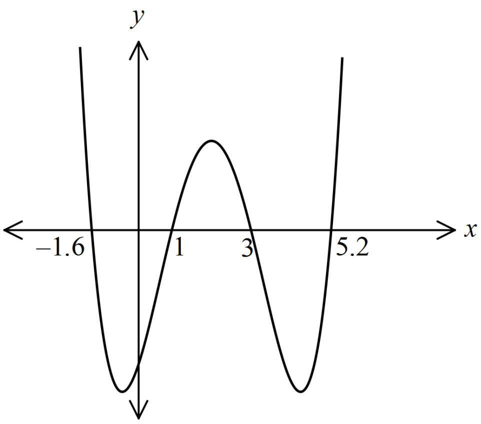
(A)



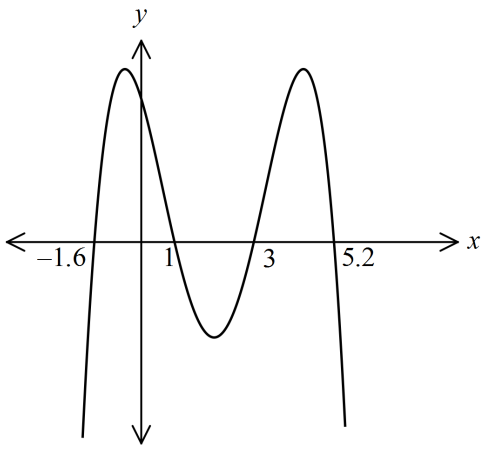
(B)



(C)



(D)



10 A particle is moving along the x-axis. The displacement of the particle at time t seconds is x metres.

At a certain time, $\dfrac{dx}{dt} = -3\,\text{m}\,\text{s}^{-1}$ and $\dfrac{d^2x}{dt^2} = 2\,\text{m}\,\text{s}^{-2}$.

Which statement describes the motion of the particle at that time?

(A) The particle is moving to the right with increasing speed.

(B) The particle is moving to the left with increasing speed.

(C) The particle is moving to the right with decreasing speed.

(D) The particle is moving to the left with decreasing speed.

Section II

90 marks
Attempt Questions 11−16
Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11−16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPERATE writing booklet.

(a) Write down the equation of a circle with centre $(-2,1)$ and radius 4. **1**

(b) Solve $|x-3| \leq 5$. **2**

(c) Rationalise the denominator of $\dfrac{4}{\sqrt{5}-\sqrt{3}}$. **2**

Give your answer in the simplest form.

(d) Evaluate $\lim_{x \to 3} \dfrac{x^3 - 27}{x-3}$. **2**

(e) Differentiate $\dfrac{x+3}{4x-5}$. **2**

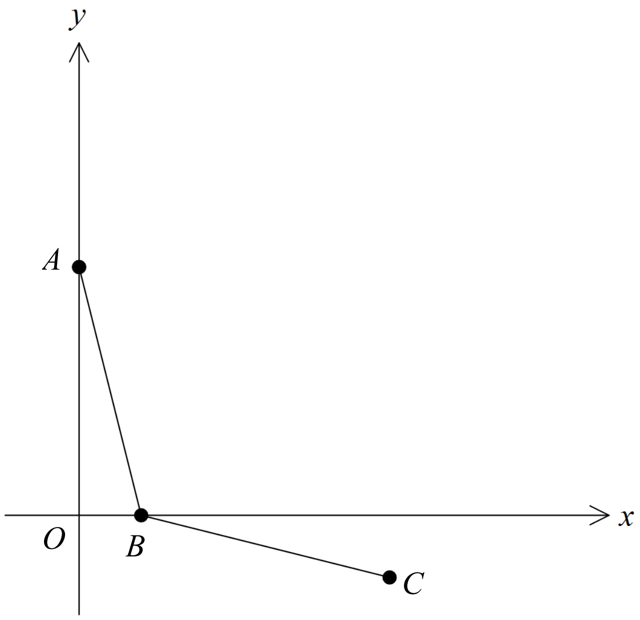
(f) Find a primitive of $\tan 3x$. **1**

(g) Find the exact value of $\displaystyle\int_0^1 \frac{x}{4+x^2}\, dx$. **2**

(h) Find the equation of the tangent to the curve $y = (2x+1)^4$ at the point $x = -1$. **3**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) The diagram below shows $A(0,4)$, $B(1,0)$ and $C(5,-1)$. The intervals AB and BC are equal in length.



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(i) Find the length of AC. **2**

(ii) Show that the equation of AC is $x+y-4=0$. **2**

(iii) Find the perpendicular distance from B to AC. **1**

(iv) Find the coordinates of D such that $ABCD$ is a rhombus. **1**

(v) Find the area of the rhombus. **2**

(b) Solve $2\sin^2 x=\sin x$ for $0\le x\le 2\pi$. **3**

(c) A parabola has focus $(3,2)$ and directrix $y=-6$.

(i) Find the focal length. **1**

(ii) Find the coordinates of the vertex. **1**

(d) For what values of k does the quadratic equation $4x^2-kx+9=0$ have no real roots? **2**

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) The third term of an arithmetic sequence is 32 and the sixth term is 17.

(i) Find the first term and the common difference. **2**

(ii) Find the sum of the first 20 terms. **1**

(b) Consider the curve $y = x^3 - 3x^2$.

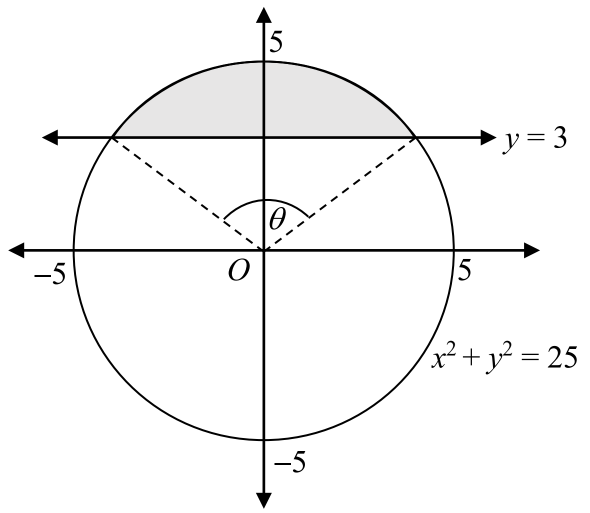
(i) Find any stationary points and determine their nature. **4**

(ii) Find the coordinates of any point(s) of inflexion. **2**

(iii) Sketch the curve labelling any stationary points, points of inflexion and x-intercepts. **2**

(iv) Find the values of x for which the curve is increasing. **1**

(c) The shaded area is enclosed by $x^2 + y^2 = 25$ and the line $y = 3$. **3**



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Find the area of the segment subtended by θ radians at the centre O.
Give your answer correct to 2 decimal places.

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Sketch the graph of $y = \frac{1}{x}$ and $x^2 = 8y$ on the same axes. **2**

(ii) Show that the coordinates of the point of intersection of the 2 curves is $(2, \frac{1}{2})$. **1**

(iii) Find the exact area of the region bounded by the x-axis and the curves from $x = 0$ to $x = 2e$. **3**

(b) A particle moves in a straight line so that its distance x metres from a fixed point O is given by $x = 2 - 3\sin 2t$ where t is measured in seconds.

(i) Where is the particle initially? **1**

(ii) Find the velocity and acceleration when $t = \frac{\pi}{3}$. **3**

(iii) When does the particle first come to rest? **1**

(iv) Find the total distance travelled by the particle in the first $\frac{\pi}{2}$ seconds. **2**

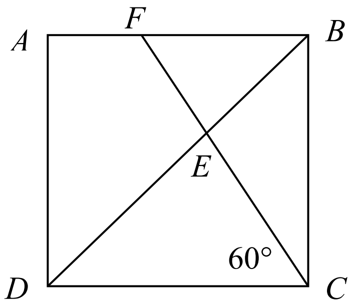
(c) An author writes a manuscript, so that on the first day he writes 54 pages, on the second day 36 pages and on each succeeding day he writes $\frac{2}{3}$ of the number of pages of the preceding day.

(i) How many pages does he write on the 5^{th} day? **1**

(ii) What is the maximum number of pages that he will write? **1**

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) A square *ABCD* of side length 1 unit is shown below. The point *F* is drawn on *AB* such that $\angle DCF = 60°$. The diagonal *DB* intersects *CF* at *E*.



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(i) Show that $\Delta DEC \parallel\!| \Delta BEF$. **3**

(ii) Show that $FB = \dfrac{1}{\sqrt{3}}$. **1**

(b) The rate of decay of radium is proportional to the mass M at that time, t, so that $\dfrac{dM}{dt} = -kM$.

Radium has a half-life of 1500 years, that is the time taken for half the initial mass to decay is 1500 years. A sample of radium begins to decay.

(i) Show that $M = M_0 e^{-kt}$ is a solution to $\dfrac{dM}{dt} = -kM$ where k and M_0 are constants. **1**

(ii) Find the exact value of k. **2**

(iii) How many years will it take for 70% of the substance to decay? **2**

(c) (i) Find $\dfrac{d}{dx}\left(e^{\cos 2x}\right)$ **1**

(ii) Hence find $\displaystyle\int x + \sin 2x\, e^{\cos 2x}\ dx$ **2**

Question 15 continues on the following page

Question 15 (continued)

(d) Peter and Ben are playing in a chess tournament. They will play two rounds and they have an equal chance of winning the first round.

If Peter wins the first round, the probability that he will win the second round is 0.7.

If Peter loses the first round, the probability that he will win the second round is 0.2.

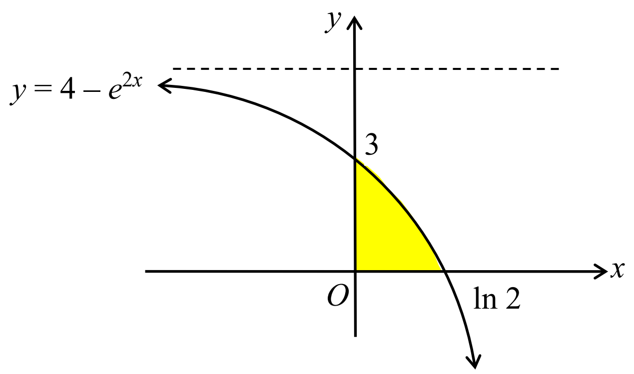
(i) Draw a probability tree to show the results of the 2 rounds of the chess tournament. **1**

(ii) Find the probability that Peter wins exactly one round. **2**

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

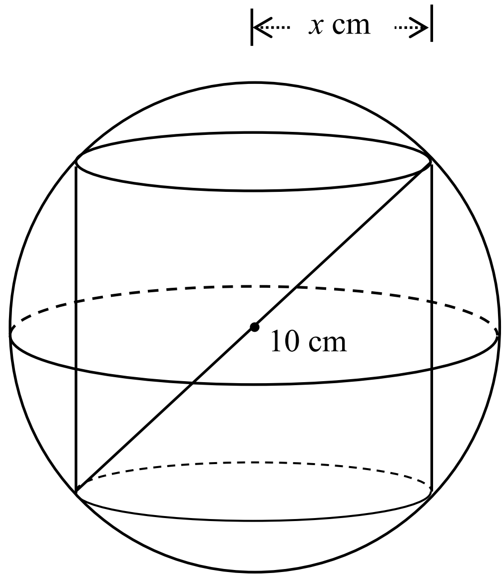
(a) The region bounded by the curve $y = 4 - e^{2x}$, the x-axis and the y-axis is rotated about the x-axis to form a solid. **3**



NOT TO SCALE

Find the exact volume of the solid.

(b) A machinist has a spherical ball made of steel with a diameter 10 cm. The ball is placed in a lathe and machined to form a cylinder.



NOT TO SCALE

(i) If the cylinder has a radius of x cm, show that the volume of the cylinder is given by **1**

$$V = \pi x^2 \sqrt{100 - 4x^2} \ \text{cm}^3$$

(ii) Find the dimensions of the cylinder that give the largest volume. Give your answer to 2 decimal places. **4**

Question 16 continues on the following page

Question 16 (continued)

(c) A farmer started a business on 1 January 2017 with 100 000 fish. His contract required him to supply 15 400 fish at a price of $10 per fish to an exporter in December each year. In the period between January and December each year, the number of fish increase by 10%.

(i) Find the number of fish just after the first period, in December 2017. **1**

(ii) Show that F_n, the number of fish just after the n th period is given by **2**

$$F_n = 154\,000 - 54\,000(1.1)^n$$

(iii) When will the farmer have sold all his fish and what will his total income be? **2**

(iv) Each December the exporter offers to buy the farmer's business by paying $15 per fish for his entire stock. If the farmer continues the process that was initially established with the exporter, when should the farmer sell to maximise his total income? **2**

End of paper

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Year 12 Mathematics Trial Examination 2017

Section I

1. $54721 \cdot 35 = 5 \cdot 5 \times 10^4$ (C)

2. $2x^2 - 5x - 9 = 0$

$a = 2,\ b = -5\quad c = -9$

$\alpha + \beta = -\frac{b}{a}$

$= -\frac{-5}{2}$

$= \frac{5}{2}$

$\alpha\beta = \frac{c}{a}$

$= \frac{-9}{2}$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{\frac{5}{2}}{\frac{-9}{2}}$

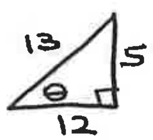
$= -\frac{5}{9}$ (C)

3. $x + 2 > 0$

$x > -2$

$\therefore$ domain $x > -2$ (D)

4. $\sin\theta = \frac{5}{13}\quad \cos\theta < 0$



$\tan\theta = -\frac{5}{12}$ (C)

5. (B)

6. $\log_{10} y = 2 - \log_{10} x$

$\log_{10} y + \log_{10} x = 2$

$\log_{10} xy = 2$

$xy = 10^2$

$y = \frac{100}{x}$ (B)

7. $P(LLL) = 1 - \frac{37}{64}$

$= \frac{27}{64}$

$P(L) = \frac{3}{4}$

$P(W) = 1 - \frac{3}{4}$

$= \frac{1}{4}$ (A)

8.

x	1	1·25	1·5	1·75	2
$y = \ln x$	ln1	ln1·25	ln1·5	ln1·75	ln2

$A \doteq \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$

$= \frac{1 \cdot 5 - 1}{6}\left[\ln 1 + 4(\ln 1 \cdot 25 + \ln 1 \cdot 75) + 2\ln 1 \cdot 5 + \ln 2\right]$

$= \frac{1}{12}\left[\ln 2 + 2\ln 1 \cdot 5 + 4(\ln 1 \cdot 25 + \ln 1 \cdot 75)\right]$ (A)

9. (D)

10 (D)

Section II

Question 11

a) $(x-h)^2+(y-k)^2=r^2$

$(x+2)^2+(y-1)^2=4^2$

$(x+2)^2+(y-1)^2=16$

b) $|x-3|\le 5$

Solve $|x-3|=5$

$x-3=5 \qquad x-3=-5$

$x=8 \qquad x=-2$

Test $x=0$ in $|x-3|\le 5$

$|0-3|\le 5$

$3\le 5$ is true

Solution $-2\le x\le 8$

c) $\dfrac{4}{\sqrt{5}-\sqrt{3}} = \dfrac{4}{\sqrt{5}-\sqrt{3}}\times\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \dfrac{4(\sqrt{5}+\sqrt{3})}{5-3}$

$= \dfrac{4(\sqrt{5}+\sqrt{3})}{2}$

$= 2(\sqrt{5}+\sqrt{3})$

d) $\lim\limits_{x\to 3}\dfrac{x^3-27}{x-3} = \lim\limits_{x\to 3}\dfrac{(x-3)(x^2+3x+9)}{(x-3)}$

$= \lim\limits_{x\to 3} x^2+3x+9$

$= 27$

e) Let $y = \dfrac{x+3}{4x-5}$

$= \dfrac{u}{v}$

where $u=(x+3) \quad v=(4x-5)$

$\dfrac{du}{dx}=1 \qquad \dfrac{dv}{dx}=4$

$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$

$= \dfrac{(4x-5)\times 1-(x+3)\times 4}{(4x-5)^2}$

$= \dfrac{4x-5-4x-12}{(4x-5)^2}$

$\therefore \dfrac{dy}{dx} = \dfrac{-17}{(4x-5)^2}$

f) $\int \tan 3x\,dx = \int \dfrac{\sin 3x}{\cos 3x}\,dx$

$= -\frac{1}{3}\int\dfrac{-3\sin 3x}{\cos 3x}\,dx$

$= -\frac{1}{3}\ln|\cos 3x|+c$

g) $\int_0^1 \dfrac{x}{4+x^2}\,dx = \frac{1}{2}\int_0^1 \dfrac{2x}{4+x^2}\,dx$

$= \frac{1}{2}\left[\ln(4+x^2)\right]_0^1$

$= \frac{1}{2}\ln(4+1)-\frac{1}{2}\ln 4$

$= \frac{1}{2}\ln\left(\frac{5}{4}\right)$

$= \frac{1}{2}\ln 1.25$

h) $y=(2x+1)^4$ at $x=-1$

$= (2(-1)+1)^4$

$= 1$

$\therefore$ point is $(-1,1)$

$\dfrac{dy}{dx} = 4(2x+1)^3\times 2$

$= 8(2x+1)^3$ at $x=-1$

$= 8(2(-1)+1)^3$

$= -8$

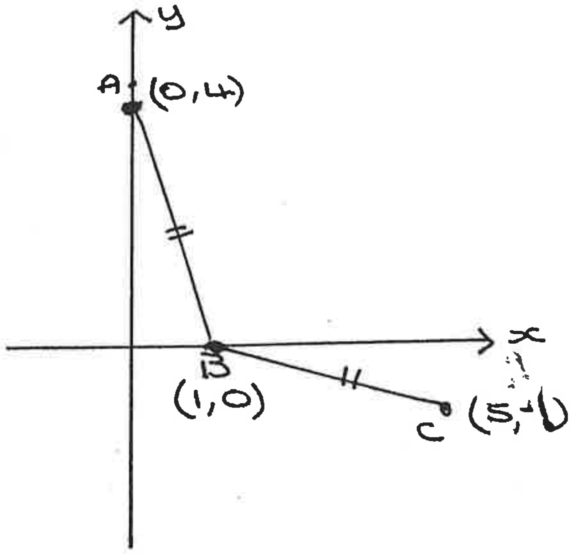
$y-y_1 = m(x-x_1)$

$y-1 = -8(x+1)$

$y-1 = -8x-8$

$y = -8x-7$

Question 12



a) (i) $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AC=\sqrt{(5-0)^2+(-1-4)^2}$

$=\sqrt{50}$

$=5\sqrt{2}$

$\therefore$ AC is $5\sqrt{2}$ units

(ii) $m=\dfrac{y_2-y_1}{x_2-x_1}$

$=\dfrac{-1-4}{5-0}$

$=-1$

$y-y_1=m(x-x_1)$

$y-4=-1(x-0)$

$y-4=-x$

$x+y-4=0$

(iii) $d=\dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$=\dfrac{|1(1)+1(0)-4|}{\sqrt{1^2+1^2}}$

$=\dfrac{3}{\sqrt{2}}$

$\therefore$ Perpendicular distance is $\frac{3}{\sqrt{2}}$

iv) $D(4,3)$

v) Area ABCD $=2\times$ Area $\triangle CBA$

$=2\times\frac{1}{2}\times5\sqrt{2}\times\frac{3}{\sqrt{2}}$

$=15$

$\therefore$ Area is 15 units^2

b) $2\sin^2 x=\sin x$

$2\sin^2 x-\sin x=0$

$\sin x(2\sin x-1)=0$

$\sin x=0$

$x=0,\pi,2\pi$

$2\sin x-1=0$

$\sin x=\frac{1}{2}$

$x=\frac{\pi}{6},\pi-\frac{\pi}{6}$

$x=\frac{\pi}{6},\frac{5\pi}{6}$

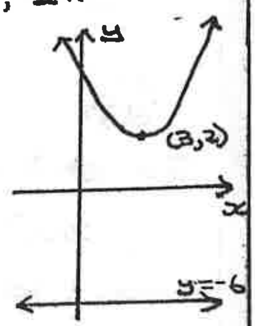
$\therefore x=0,\frac{\pi}{6},\frac{5\pi}{6},\pi,2\pi$

c) i) $2a=8$

$a=4$

focal length is 4

ii) vertex $(3,-2)$



d) $4x^2-kx+9=0$

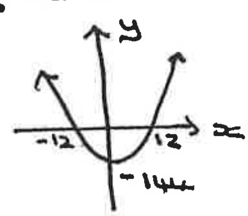
$\Delta=b^2-4ac$

$=(-k)^2-4\times4\times9$

$=k^2-144$

For no real roots $\Delta<0$

$k^2-144<0$



$\therefore -12<k<12$

Question 13

a) i) $T_3 = a + 2d = 32$ —— ①
$T_6 = a + 5d = 17$ —— ②

① − ② $\quad -3d = 15$
$d = -5$

sub ① $\quad a + 5d = 17$
$a + 5(-5) = 17$
$a = 42$

∴ first term is 42 and common difference is −5

ii) $S_n = \frac{n}{2}[2a + (n-1)d]$
$S_{20} = \frac{20}{2}[2(42) + (20-1)(-5)]$
$= -110$

b) $y = x^3 - 3x^2$
$\frac{dy}{dx} = 3x^2 - 6x$
$\frac{d^2y}{dx^2} = 6x - 6$

For stationary points $\frac{dy}{dx} = 0$
$3x^2 - 6x = 0$
$3x(x-2) = 0$
$x = 0, 2$

__when $x = 0$__
$y = 0$

Test the nature
$\frac{d^2y}{dx^2} = 6x - 6$ when $x = 0$
$= 6(0) - 6$
$= -6$

As $\frac{d^2y}{dx^2} < 0$ the curve is concave down
∴ $(0,0)$ is a local maximum

__when $x = 2$__
$y = x^3 - 3x^2$
$= 2^3 - 3(2)^2$
$= -4$

∴ $(2,-4)$ is a stationary point

Test the nature
$\frac{d^2y}{dx^2} = 6x - 6$ at $x = 2$
$= 6(2) - 6$
$= 6$

As $\frac{d^2y}{dx^2} > 0$ the curve is concave up
∴ $(2,-4)$ is a local minimum

ii) For points of inflexion $\frac{d^2y}{dx^2} = 0$ and there is a change in concavity.

$\frac{d^2y}{dx^2} = 6x - 6$
$6x - 6 = 0$
$6x = 6$
$x = 1$

when $x = 1$,
$y = x^3 - 3x^2$
$= 1^3 - 3(1)^2$
$= -2$

∴ $(1,-2)$ may be a point of inflexion

__check concavity__

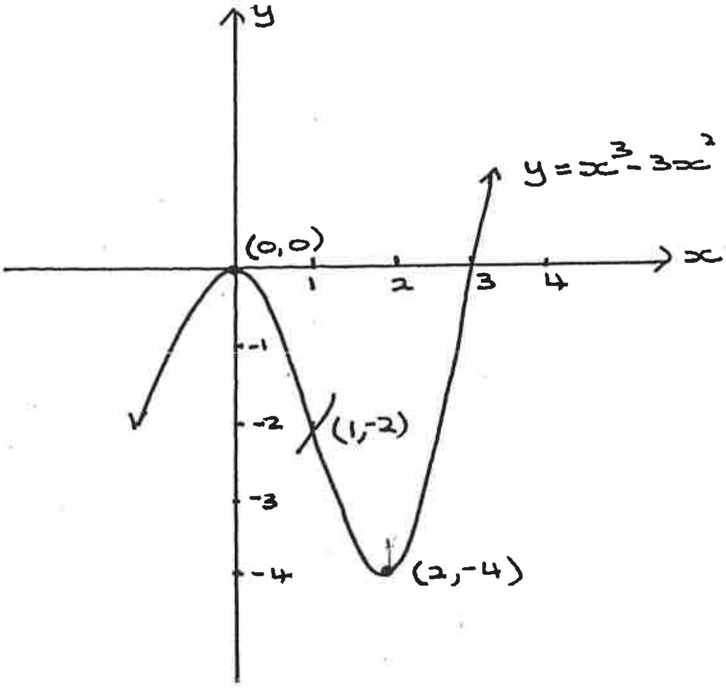
	0.9		1.1
x	1^-	1	1^+
$\frac{d^2y}{dx^2} = 6x - 6$	−0.6	0	+0.6

As there is a change in concavity $(1,-2)$ is a point of inflexion

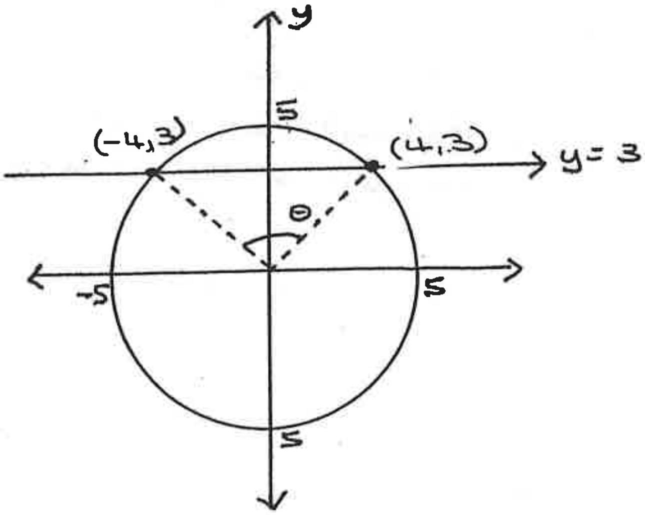
iii) $y = x^3 - 3x^2$

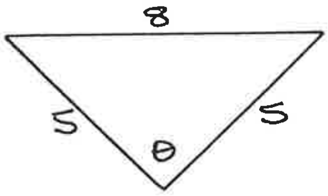
$= x^2(x-3)$

$\therefore$ x-intercepts are $(0,0)$ and $(3,0)$



iv) The curve is increasing for $x < 0$, $x > 2$





$\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

$\cos C = \dfrac{5^2+5^2-8^2}{2\times5\times5}$

$= \dfrac{-14}{50}$

$\hat{C} \doteqdot 1.854590436$

$= 1.85$ to 2dp

$A = \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2}\times5^2(1.85 - \sin 1.85)$

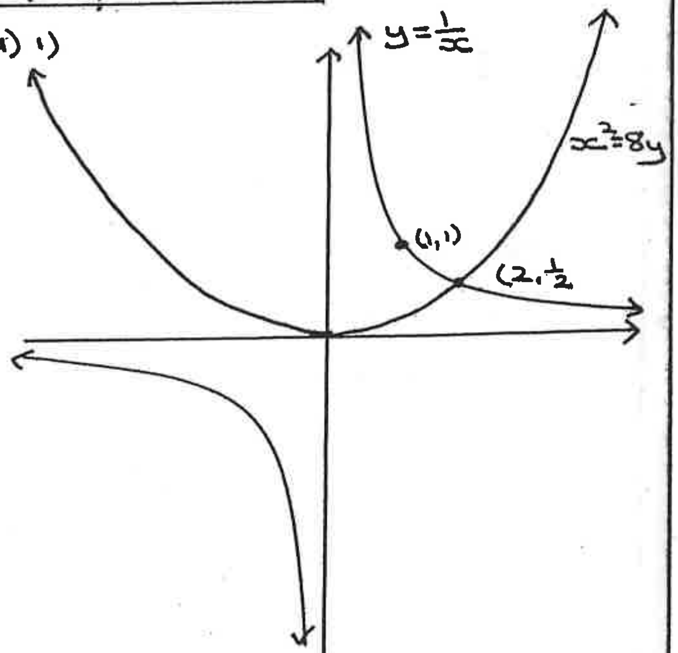
$\doteqdot 11.18238045$

$= 11.18$ to 2dp

$\therefore$ Area is 11.18 units^2 to 2dp

Question 14

a) i)



ii) $y = \frac{1}{x}$ —— ①

$x^2 = 8y$ —— ②

sub ① in ②

$x^2 = \frac{8}{x}$

$x^3 = 8$

$x = 2$

sub ① $y = \frac{1}{x}$

$= \frac{1}{2}$

$\therefore$ point of intersection is $(2, \frac{1}{2})$

iii) $A = \int_0^2 \frac{x^2}{8}\,dx + \int_2^{2e} \frac{1}{x}\,dx$

$= \left[\frac{x^3}{24}\right]_0^2 + \left[\ln x\right]_2^{2e}$

$= \left[\frac{2^3}{24}\right] - \left[\frac{0}{24}\right] + \left[\ln 2e\right] - \left[\ln 2\right]$

$= \frac{8}{24} - 0 + \ln 2e - \ln 2$

$= \frac{1}{3} + \ln 2 + \ln e - \ln 2$

$= 1\frac{1}{3}$

$\therefore$ Area is $1\frac{1}{3}$ units^2

b) $x = 2 - 3\sin 2t$

i) $x = 2 - 3\sin 2(0)$

$= 2$

$\therefore$ The particle is 2m to the right

ii) $\dot{x} = -6\cos 2t$ at $x = \frac{2\pi}{3}$

$= -6\cos\left(2 \times \frac{2\pi}{3}\right)$

$= 3$

$\therefore$ velocity is 3m/s

$\ddot{x} = 12\sin 2t$ at $x = \frac{2\pi}{3}$

$= 12\sin\left(2 \times \frac{2\pi}{3}\right)$

$= 12\frac{\sqrt{3}}{2}$

$= 6\sqrt{3}$

$\therefore$ acceleration is $6\sqrt{3}$ m/s^2

iii) at rest, $\dot{x} = 0$

$\dot{x} = -6\cos 2t$

$-6\cos 2t = 0$

$2t = \frac{\pi}{2}, \frac{3\pi}{2}$

$t = \frac{\pi}{4}, \frac{3\pi}{4}$

$\therefore$ particle first comes to rest at $t = \frac{\pi}{4}$

iv) when $t = 0$ $\quad x = 2$

$t = \frac{\pi}{4}$ $\quad x = 2 - 3\sin 2t$

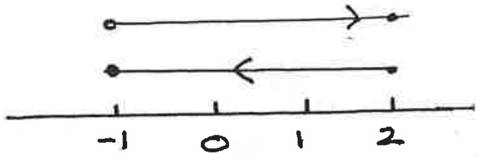
$= 2 - 3\sin\left(2 \times \frac{\pi}{4}\right)$

$= -1$

$t = \frac{\pi}{2}$ $\quad x = 2 - 3\sin 2t$

$= 2 - 3\sin\left(2 \times \frac{\pi}{2}\right)$

$= 2$



distance is 6 metres

c) i) $a = 54 \quad r = \frac{2}{3}$

$T_n = ar^{n-1}$

$T_5 = 54 \times \left(\frac{2}{3}\right)^{5-1}$

$= 54 \times \left(\frac{2}{3}\right)^4$

$= 10\frac{2}{3}$

$\therefore$ He writes $10\frac{2}{3}$ pages

ii) $S_\infty = \frac{a}{1-r}$

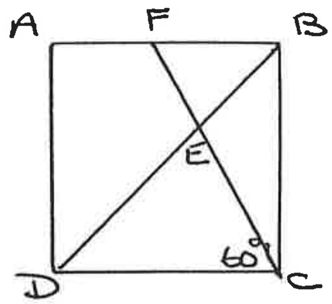
$= \frac{54}{1-\frac{2}{3}}$

$= 162$

$\therefore$ He will write a maximum of 162 pages

Question 15

a)



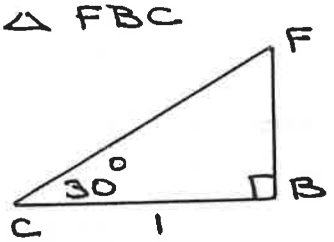
i) In $\triangle DEC$ and $\triangle BEF$

$D\hat{E}C = B\hat{E}F$ — vertically opposite angles are equal

$E\hat{D}C = E\hat{B}F$ — alternate angles are equal $AB \parallel DC$. $AB \parallel DC$ as ABCD is a square. Opposite sides of a square are parallel.

$\therefore \triangle DEC \parallel\!\mid \triangle BEF$ equiangular

ii) In $\triangle FBC$



$F\hat{B}C = 90°$ — ABCD is a square. All angles of a square are 90°

$F\hat{C}B + D\hat{E}C = 90°$ — ABCD is a square. All angles of a square are 90°

$F\hat{C}B = 30°$

$\frac{FB}{CB} = \tan 30°$

$\frac{FB}{1} = \tan 30°$

$FB = \frac{1}{\sqrt{3}}$

b) i) $M = M_0 e^{-kt}$

$\frac{dM}{dt} = M_0 \times -ke^{-kt}$

$= -kM_0 e^{-kt}$

$\therefore \frac{dM}{dt} = -kM$

ii) $M = M_0 e^{-kt}$

$\frac{1}{2}M_0 = M_0 e^{-1500k}$

$\frac{1}{2} = e^{-1500k}$

$\ln(\frac{1}{2}) = \ln(e^{-1500k})$

$-1500k \ln e = \ln(\frac{1}{2})$

$-1500k = \ln 2^{-1}$

$-1500k = -\ln 2$

$k = \frac{\ln 2}{1500}$

iii) $M = M_0 e^{-kt}$

$0.3M_0 = M_0 e^{-kt}$

$0.3 = e^{-\frac{\ln 2}{1500}t}$

$\ln(0.3) = \ln\left(e^{\frac{-\ln 2}{1500}t}\right)$

$\ln(0.3) = \frac{-\ln 2}{1500}t$

$t = \frac{\ln(0.3)}{\left(-\frac{\ln 2}{1500}\right)}$

$t \doteqdot 2605.448391$

$\therefore$ It will take 2605 years

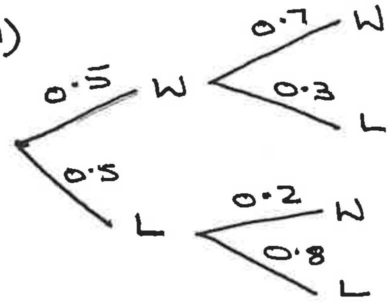
c) i) $\frac{d}{dx}\left(e^{\cos 2x}\right) = -2\sin 2x\, e^{\cos 2x}$

ii) $\int x + \sin 2x e^{\cos 2x} \, dx$

$= \int x \, dx - \frac{1}{2}\int -2\sin 2x e^{\cos 2x} \, dx$

$= \frac{x^2}{2} - \frac{1}{2}e^{\cos 2x} + C$

d)



$P(WL, LW) = 0{\cdot}5 \times 0{\cdot}3 + 0{\cdot}5 \times 0{\cdot}2$
$= 0{\cdot}25$

$\therefore$ Probability Peter wins exactly one round is 0·25

Question 16

a) $y = 4 - e^{2x}$

$y^2 = (4 - e^{2x})^2$

$= 16 - 8e^{2x} + e^{4x}$

$V = \pi \int y^2 \, dx$

$= \pi \int_0^{\ln 2} (16 - 8e^{2x} + e^{4x}) \, dx$

$= \pi \left[16x - 4e^{2x} + \frac{1}{4} e^{4x} \right]_0^{\ln 2}$

$= \pi \left[16\ln 2 - 4e^{2\ln 2} + \frac{1}{4} e^{4\ln 2} \right] - \left[0 - 4e^{0} + \frac{1}{4} e^{0} \right]$

$= \pi \{ [16\ln 2 - 16 + 4] - [0 - 4 + \frac{1}{4}] \}$

$= \pi [16\ln 2 - 12] - [-3\frac{3}{4}]$

$= (16\ln 2 - \frac{33}{4})\pi$

$\therefore$ Volume is $(16\ln 2 - \frac{33}{4})\pi$

b) Let the height of the cylinder be h

$h^2 + (2x)^2 = 10^2$

$h^2 = 100 - 4x^2$

$h = \sqrt{100 - 4x^2}$

$V = \pi r^2 h$

$\therefore V = \pi x^2 \sqrt{100 - 4x^2}$

ii) $V = \pi x^2 (100 - 4x^2)^{\frac{1}{2}}$

$= u \; v$

where

$u = \pi x^2 \qquad v = (100 - 4x^2)^{\frac{1}{2}}$

$\frac{du}{dx} = 2\pi x \qquad \frac{dv}{dx} = \frac{1}{2}(100 - 4x^2)^{-\frac{1}{2}} \times -8x$

$= -4x(100 - 4x^2)^{-\frac{1}{2}}$

$\frac{dV}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

$= 2\pi x (100 - 4x^2)^{\frac{1}{2}} - 4\pi x^3 (100 - 4x^2)^{-\frac{1}{2}}$

$= 2\pi x (100 - 4x^2)^{-\frac{1}{2}} [100 - 4x^2 - 2x^2]$

$= 2\pi x (100 - 4x^2)^{-\frac{1}{2}} (100 - 6x^2)$

$= \frac{2\pi x (100 - 6x^2)}{\sqrt{100 - 4x^2}}$

For stationary points $\frac{dV}{dx} = 0$

$\frac{2\pi x (100 - 6x^2)}{\sqrt{100 - 4x^2}} = 0$

$2\pi x (100 - 6x^2) = 0$

$4\pi x (50 - 3x^2) = 0$

$x = 0, \pm \sqrt{\frac{50}{3}} \quad x > 0$

$\therefore x = \sqrt{\frac{50}{3}}$

$= \frac{5\sqrt{2}}{\sqrt{3}}$

$= \frac{5\sqrt{6}}{3}$

$\doteqdot 4{\cdot}082482905$

$x = 4{\cdot}08$ to 2dp

when $x = \frac{5\sqrt{6}}{3}$

$h = \sqrt{100 - 4\left(\frac{5\sqrt{6}}{3}\right)^2}$

$= \sqrt{\frac{100}{3}}$

$\doteqdot 5{\cdot}773502692$

$h = 5{\cdot}78$ to 2dp

∴ The radius is 4·08 cm and the height is 5·78 cm correct to 2dp.

Test the nature

	4		4·1
x	$4{\cdot}08^-$	4·08	$4{\cdot}08^+$
$\frac{dv}{dx}$	16·76		−3·87

∴ The largest volume occurs when the radius is 4·08 cm and the height is 5·78 cm

c) i) Number of fish $= 100000(1{\cdot}1) - 15400$

$= 94600$

∴ Number fish after the first period is 94600

ii) Let F_n be the number of fish after the n^{th} period

$F_1 = 100000(1{\cdot}1) - 15400$

$F_2 = F_1(1{\cdot}1) - 15400$

$= [100000(1{\cdot}1) - 15400]1{\cdot}1 - 15400$

$= 100000(1{\cdot}1)^2 - 15400(1{\cdot}1) - 15400$

$= 100000(1{\cdot}1) - 15400(1 + 1{\cdot}1)$

$F_3 = F_2(1{\cdot}1) - 154000$

$= [100000(1{\cdot}1)^2 - 15400(1 + 1{\cdot}1)](1{\cdot}1) - 15400$

$= 100000(1{\cdot}1)^3 - 15400(1 + 1{\cdot}1 + 1{\cdot}1^2)$

continuing the pattern

$F_n = 100\,000(1{\cdot}1)^n - 15400(1 + 1{\cdot}1 + \dots + 1{\cdot}1)^n$

$= 100000(1{\cdot}1)^n - 15400\left[\frac{1((1{\cdot}1)^n - 1}{1{\cdot}1 - 1}\right]$

$= 100000(1{\cdot}1)^n - 154000(1{\cdot}1)^n + 154000$

$F_n = 154000 - 54000(1{\cdot}1)^n$

iii) If the farmer has sold all his fish $F_n = 0$

$154000 - 54000(1{\cdot}1)^n = 0$

$54000(1{\cdot}1)^n = 154000$

$(1{\cdot}1)^n = \frac{154000}{54000}$

$\ln(1{\cdot}1^n) = \ln\left(\frac{77}{27}\right)$

$n = \frac{\ln\left(\frac{77}{27}\right)}{\ln 1{\cdot}1}$

$\doteqdot 10{\cdot}99534759$

∴ He would have sold all his fish after 11 years

Total income $= 11 \times 15400 \times 10$

$= \$169400$

∴ His total income is $169400

(iv) Total income, I_n, after n harvests $= \$15 \times F_n + \$10 \times 15400 \times n$

$= 15[154000 - 54000(1{\cdot}1)^n + 154000n$

$I_n = 2310000 - 810000(1{\cdot}1)^n + 154000n$

By trial an error

when $n = 6$

$I_6 = 2310000 - 810000(1{\cdot}1)^6 + 154000 \times 6$

$= 1799035{\cdot}59$

when $n = 7$

$I_7 = 2310000 - 810000(1{\cdot}1)^7 + 154000 \times 7$

$= 1809539{\cdot}149$

when $= 8$

$I_8 = 2310000 - 810000(1{\cdot}1)^8 + 154000 \times 8$

$= 1805693{\cdot}064$

∴ The farmer should sell after the 7^{th} harvest